Løsningsskitse:

9.1 1. $\mathbf{r}_u(u,v) = [1,1,0]$, $\mathbf{r}_v(u,v) = [0,2v,1]$. $\mathbf{r}_u(0,0) = [1,1,0]$, $\mathbf{r}_v(0,0) = [0,0,1]$. $(\mathbf{r}_u \times \mathbf{r}_v)(0,0) = [1,-1,0]$. Tangentplanen har ligningen $0 = [x,y,z-1]\cdot[1,-1,0] \Leftrightarrow y = x$.

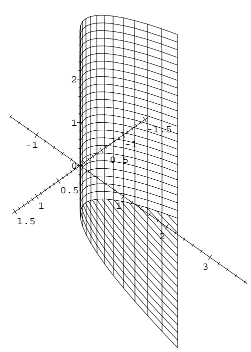
2. $E = 2$, $F = 2v$, $G = 4v^2+1$.

3. "uden": $\mathbf{x}(t) = [t, t+t^2, t+1], \mathbf{x}'(t) = [1, 1+2t, 1], |\mathbf{x}'(t)| = \sqrt{4t^2+4t+3}$.
"med": $|\mathbf{x}'(t)| = \sqrt{E(t,t)u'(t)^2 + 2F(t,t)u'(t)v'(t) + G(t,t)v'(t)^2} =$

$\sqrt{2+2\cdot 2t+4t^2+1} = \sqrt{4t^2+4t+3}$.

4. $A = \int_{-1}^{1}\int_{-1}^{1}\sqrt{EG-F^2}du\,dv = \int_{-1}^{1}\int_{-1}^{1}\sqrt{4v^2+2}du\,dv = 2\int_{-1}^{1}\sqrt{4v^2+2}dv =$
$[v\sqrt{4v^2+2} + \ln(\sqrt{2}v + \sqrt{2v^2+1}]_{-1}^{1} =$
$2\sqrt{6} + \ln(\sqrt{2}+\sqrt{3}) - \ln(-\sqrt{2}+\sqrt{3}) = 2\sqrt{6} - 2\ln(\sqrt{3}-\sqrt{2}) \simeq$
7.19.

5. Elimineres u og v fås $y = x + (z-1)^2$. Fladen S er en parabolsk cylinder.



Figur 1: En parabolsk cylinder

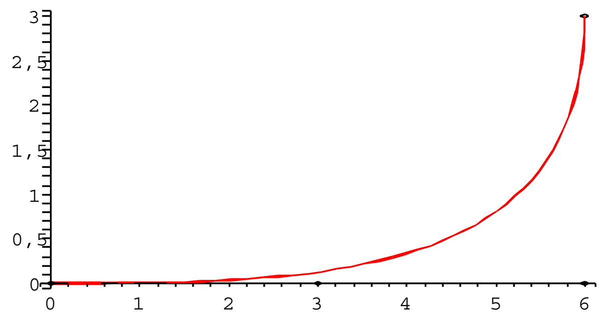
9.2 Med Bernsteinpolynomierne $B_{i,3}(t)$ – se [JR] eller slides fra 6. lektion – fås:

$$B(t) = B_{0,n}(t)P_0 + B_{1,n}(t)P_1 + B_{2,n}(t)P_2 + B_{3,n}(t)P_3 = [-3t^3+9t, 3t^3].$$

Se s. 2 og sammenlign med figuren for den kubiske spline gennem punkterne P_i fra 8. lektion!

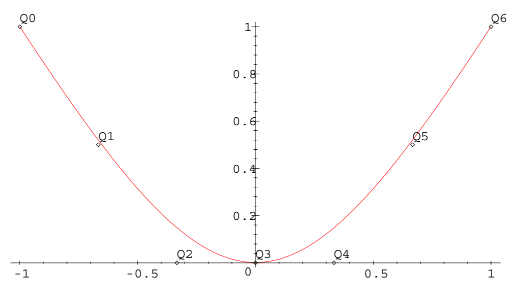
9.3 1. Check, at $\mathbf{A}_3\begin{bmatrix}\mathbf{v}_0\\ \mathbf{v}_1\\ \mathbf{v}_2\end{bmatrix} = 3\begin{bmatrix}\overrightarrow{P_0P_1}\\ \overrightarrow{P_0P_2}\\ \overrightarrow{P_1P_2}\end{bmatrix}$.

2. Parablen gennem de 3 punkter har parameterfremstilling $\mathbf{r}(t) = [t, t^2]$ og dermed tangenter $\mathbf{r}'(-1) = [1,-2]$ i P_0, hhv. $\mathbf{r}'(-1) = [1,2]$ i P_1. De er ikke parallele til $\mathbf{v}_0$, hhv. $\mathbf{v}_2$. Se s. 2.

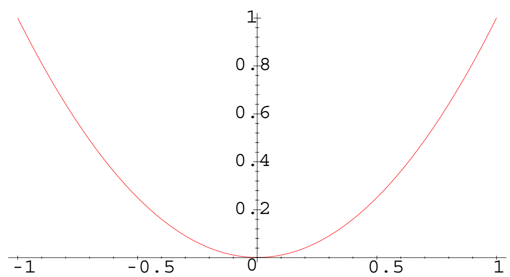


Figur 2: Bezierkurve

3. $Q_0 = P_0$, $\overrightarrow{OQ_1} = \overrightarrow{OP_0} + \frac{1}{3}\mathbf{v}_0 = [-1+\frac{1}{3}, 1-0.5] = [\frac{-2}{3}, 0.5]$,
$\overrightarrow{OQ_2} = \overrightarrow{OP_1} - \frac{1}{3}\mathbf{v}_1 = -[\frac{1}{3}, 0]$, $Q_3 = P_1$, $\overrightarrow{OQ_4} = \overrightarrow{OP_1} + \frac{1}{3}\mathbf{v}_1 = [\frac{1}{3}, 0]$,
$\overrightarrow{OQ_5} = \overrightarrow{OP_2} - \frac{1}{3}\mathbf{v}_2 = [1,1] - [\frac{1}{3}, 0.5] = [\frac{2}{3}, 0.5]$.



Figur 3: Kubisk spline gennem 3 punkter



Figur 4: Parabel gennem de samme 3 punkter