General remarks

The assessment of the course will be through two exercises which we will ask you to work on in groups of two or three and hand in. One will be posed this week and another next week. More about that later.

Manifolds

When? Monday, September 13; 8:45 – 11:45
Where? Fredrik Bajers Vej 7G5-109

Lectures

Aims and Content

Manifolds are, on the one hand geometric objects, that one can "see" in the lower dimensional cases, and on the other hand the general definition may seem way too complicated to start with.

The underlying idea is, that we want enough structure to be able to define smooth functions and their differentials, and we want to define concepts which are coordinate invariant, i.e., in physics, the energy should not depend on whether we do calculations in polar coordinates or in rectangular coordinates.

We define topological manifolds first. A topological manifold is a space which localy "looks like" a Euclidean space, $\mathbb{R}^n$. A sphere looks like a plane locally, but not globally. The resemblance to the plane is via maps, like the charts we all use, when navigating on the globe. On a topological manifold, we can take limits of sequences, and continuous functions are defined.

Smooth, or C^∞, manifolds are topological manifolds with extra structure, an atlas of smooth charts.

A topological manifold may have several different smooth structures or none at all, but this happens in dimension at least 4, and the math behind it is extremely complicated.

Lecturer:

Lisbeth Fajstrup

References:

[LWT] Ch. 5. Appendix A1-A5. We will not spend much time dwelling on the concept of a topological space. A manifold has an underlying topological space, which is required to be Hausdorff and second countable, and hence we do need to understand the definition of this. But please do not get lost in the details here.

Exercises:

- p.54 ex.5.3
- Prove that the graph of the function $f(t) = t^{2/3}$ is a smooth manifold. Hint: You can cover it with just one chart.

- Prove that the charts provided in Proposition 5.17 are in fact charts on $M \times N$. (You may need to check the definition of the topology on a product from section A6). Then read section A6 and prove that $M \times N$ is a topological manifold. If you have the time, prove that the charts are smoothly compatible.

Smooth Maps on a Manifold

When? Monday, September 15; 12:30 – 15:30
Where? Fredrik Bajers Vej 7G5-109

Lectures

Aims and Content

We look closely at functions defined on smooth manifolds and maps between manifolds. The trick of this lecture is to use the local Euclidean property of a manifold. In short, we define maps locally in coordinates. By mathematical magic, all local representations are equivalent. More precisely, we compose the maps on manifolds with coordinate charts to get new functions defined on open sets of the Euclidean space. Now, a function f is said to be smooth at a point p if its composition with the inverse of a coordinate chart say (U, ϕ) (with $p \in U$) is smooth in the ordinary sense as a function defined on an open set of $\mathbb{R}^n$. In this way a whole bunch of results from the calculus courses can be applied locally, e.g. inverse function and implicit function theorems. But don't worry: We will not assumed that you still remember or know them. By the way, in Appendix B1 and B2, you can read about them.

Lecturer:

Rafael Wisniewski

References:

[LWT] Ch. 6. Appendix B1-B2.

Exercises:

- Problem 6.1, pp. 62
- Problem 6.2. pp. 62
- Problem 6.7. pp. 62

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