GENERAL INFORMATION

See the course web page `http://www.math.aau.dk/ raussen/PHDK/10F/` and the welcome page `http://www.math.aau.dk/ raussen/PHDK/10F/welcome.pdf` including information on the structure of the course.

Litterature.

Main Reference. [HSD]: M.W. Hirsch, S. Smale, R.L. Devaney, *Dynamical Systems, Differential Equations & an Introduction to Chaos*, 2nd ed., Elsevier, 2004.
Bob Devaney has a web page with corrections on display.

Supplementary reading. See again the course web page
`http://www.math.aau.dk/ raussen/PHDK/10F/`.

Preparation. We will start lectures along with chapter 3 of the book. Hence we expect you to have worked your way through the first two chapters beforehand. You should not dwell too long on section 1.5 on the Poincaré map; if you do, make sure to correct the differentiation of f with respect to x_0 to differentiation with respect to φ as you may have seen from the list of corrections on the homepage for the book.

Exercise sessions. Every day will include two exercise session. We would like to ask you to participate very actively in these. The examples treated and the reasoning asked of you will make the lectures more concrete and they will test your understanding of the material. Please bring a laptop that can help with illustrations or calculations along with the exercises.

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INTRODUCTION. SIMPLE 2D LINEAR DYNAMICAL SYSTEMS

Tue, March 2; 9 – 11:45
Fredrik Bajers Vej 7E, room 3-109.

Startup. Welcome to the course. General information. Course overview. What you can expect. What we do expect.

Lectures.

Aims and Content. In chapter 2 of [HSD], explicit solutions of a linear planar system $X' = AX$ were constructed using eigenvalues and eigenvectors of A. Specific solutions were combined to yield all solutions to such a system; at least in the case when the eigenvalues are real; cf. the Theorem on p. 35. In fact, a similar analysis is possible for complex eigenvalues. The qualitative behaviour of the solutions depends on the eigenvalues - are they real or complex, what are their signs (signs of their real parts) or are they zero? To get an idea of the qualitative behaviour, we will study systems with three types of matrices,

$$A = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}, A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$$

$$A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

and the associated phase portraits in detail.

Lecturer: Lisbeth Fajstrup

References: [HSD] 3.1, 3.2 and 3.3.

Exercises:

- **HSD, ch. 2** p. 37, Ex. 2, 3.
- **HSD, ch. 3** p. 57, Ex. 1; use the solver referred to on the course webpage.

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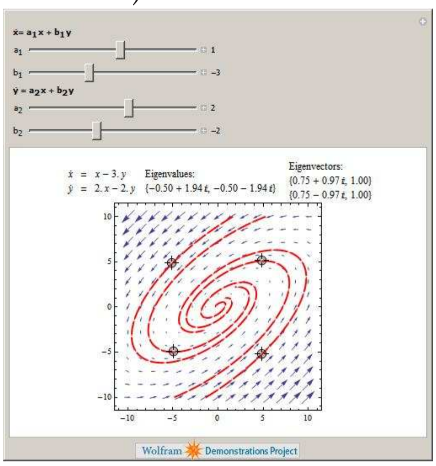
Behaviour and classification of 2D linear dynamical systems.

Tue, March 2; 12:30 – 15:15

Lectures.

Aims and Content. Dynamical sytems associated to general invertible (2×2)-matrices show the same performance (look-alike phase planes) as those explained in the previous standard examples. This can be seen by conjugation of a general such matrix to a canonical matrix by "change of coordinates" – this concept will be explained in a crash course at the beginning.
The associated dynamical systems are then characterized by

- sinks, sources or saddles in the case of two independent eigenvectors associated to real eigenvalues;
- spiral sinks or sources in the case of two (conjugate) complex eigenvalues (e.g., in the case of a harmonic oscillator)



- a transition case that occurs with an eigenvalue of algebraic multiplicity two with a one-dimensional associated eigenspace.

It turns out, that the trace and the determinant of a (2×2)-matrix in common determine which type of dynamical system the matrix describes. These two characteristic numbers can be plotted in a 2D-diagram; each point in that diagram corresponds to a particular phase diagram. Crossing the axes $\det A = 0$, $\mathrm{tr}\ A = 0$ and the curve $\Delta A = 0$ (Δ: discriminant) in that diagram gives rise to bifurcations, i.e., abrupt changes in the associated phase planes. See Figure 4.1 on p. 63 of the textbook and the table on the following page.

Lecturer: Martin Raussen

References:

HSD: ch. 3.4 & 4.1
Wikipedia: Matrix differential equation

Exercises:

- **HSD, ch. 3** Remaining parts of 1, 2(i)(iii)(v), 4.

2D LINEAR ODES

Trace, determinant, equilibria.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

trace: $\mathrm{tr}(A) = a + d = \lambda_1 + \lambda_2$
determinant: $\det(A) = ad - bc = \lambda_1\lambda_2$
discriminant: $\Delta(A)=\mathrm{tr}(A)^2 - 4\det(A)$

$\det(A) < 0$ $\Rightarrow \Delta(A) > 0$	$\lambda_1 < 0 < \lambda_2$ λ_i real	saddle point
$\det(A) > 0$ $\Delta(A) > 0$ $\mathrm{tr}(A) > 0$	λ_i real, same sign $\lambda_i > 0$	source
$\mathrm{tr}(A) < 0$	$\lambda_i < 0$	drain
$\Delta(A) < 0$ $\mathrm{tr}(A) > 0$	λ_i complex $\mathrm{Re}(\lambda_i) > 0$	spiral source
$\mathrm{tr}(A) < 0$	$\mathrm{Re}(\lambda_i) < 0$	spiral sink
$\mathrm{tr}(A) = 0$	λ_i imaginary	center
$\Delta(A) = 0$	real eigenvalue of multiplicity 2	bifurcation or linear source/drain
$\det(A) = 0$ $\mathrm{tr}(A) > 0$	one eigenvalue is 0 2nd eigenvalue > 0	equilibrium line source
$\mathrm{tr}(A) < 0$	2nd eigenvalue > 0	equilibrium line drain
$\mathrm{tr}(A) = 0$	$\lambda_i = 0$	all points equilibria or parallel invariant lines

TABLE 1. Invariants, eigenvalues, equilibria

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