STABILITY AND LYAPUNOV FUNCTIONS

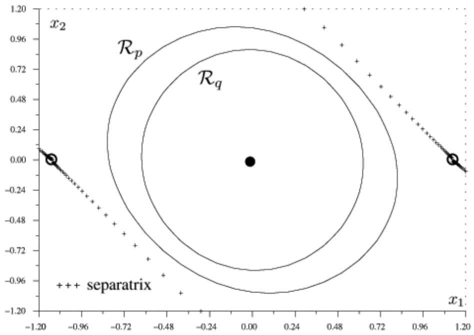

Figure 3: Estimates of SR: $\mathcal{R}_q$ and $\mathcal{R}_p$.

March 8, 9-11:45
Niels Jernes Vej 14, room 3-119.

Lectures.

Aims and Content. Stability of a dynamical system can be given (at least) two meanings:

1. An equilibrium point may be stable, asymptotically stable or unstable. We saw earlier, that for hyperbolic equilibrium points is is enough to look at the linearization to decide about stability close to this point. For non-hyperbolic points, one has to use more sophisticated tools. One such tool is the use of Lyapunov functions.

Lecturer: Lisbeth Fajstrup

References:

HSD: 8.4, 9.1, 9.2.
Wikipedia: Lyapunov function

Exercises:

HSD pp. 211/212:
Read some of the examples in 9.1; 9.2 and fill in details.

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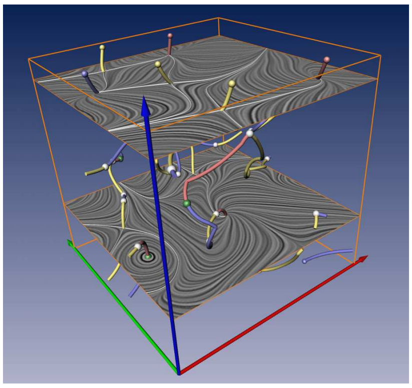
STABILITY AND BIFURCATIONS



March 8, 12:30 – 15:15

Lectures.

Aims and Content. 2. One may also consider stability of the system as such. What are the characteristic features of the system behaving under variation of the parameters describing it? Varying for instance the setup of an experiment gives rise to a variation of the parameters in the system, and hence a family of systems $X' = F_a(X)$, where a represents the varying parameter. A bifurcation occurs at $a = a_0$, if the behaviour of the system is significantly different for values $a_0 - \varepsilon$ and values $a_0 + \varepsilon$.

Lecturers: Lisbeth Fajstrup and Martin Raussen

References:
HSD: 8.5.
Wikipedia: Bifurcation theory

Demos on the internet: Bifurcation diagrams

Exercises: The behaviour of a non-linear system off and inbetween the equilibrium points is the theme of this exercise.

(1) The system
$$x' = x^2 - 1, \qquad y' = -xy$$
has two saddle points (where?) Prove that there is a solution curve which is a stable curve for one saddle point and an unstable curve for the other. [1]

(2) The system
$$x' = -2x(x-1)(2x-1), y' = -2y$$
has three equilibrium points. Classify the linearization at these three points and use a plot tool to see how the solutions behave (at large).

(3) Same question for the system
$$x' = y, \qquad y' = -x^3 + x.$$
Here, the unstable curves of the saddle point turn around and come back as the stable curves - they are homoclinic orbits.[2]

[1] Hint: It moves along the x-axis.
[2] Why closed orbits? Consider level curves of the function $2y^2 - 2x^2 + x^4$.

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