LISBETH FAJSTRUP
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GRADIENT SYSTEMS. HAMILTONIAN SYSTEMS

March 10, 9-11:45
Fredrik Bajers Vej 7D, room 2-109.

Lectures.

Aims and Content. Besides linear differential equations there are two classes of dynamical systems that are particularly regular: gradient systems and Hamiltonian systems. A common feature of these is that they can be efficiently analyzed by studying an associated function: a Lyapunov function for the first class and a Hamiltonian function for the latter.

In particular, an isolated minimum of a Lyapunov function is an equilibrium point for a gradient system. During the lecture we will introduce the notion of a regular value of a function. If c is the regular value of a function V then the gradient vector field $\mathrm{grad}(V)$ is perpendicular to the level set $V^{-1}(c)$.

Conservative mechanical systems are described by Hamiltonian systems. A Hamiltonian function is a constant of motion - it is constant along every solution of the system.

Lecturer: Rafael Wisniewski

References:

HSD: ch. 9.3-4.
Wikipedia: Hamiltonian mechanics

[HSD],

Exercises:

Bifurcations: [HSD], p. 185-6, exc. 5.
Gradient/Hamiltonian: [HSD], p. 213, exc. 10 and 11.

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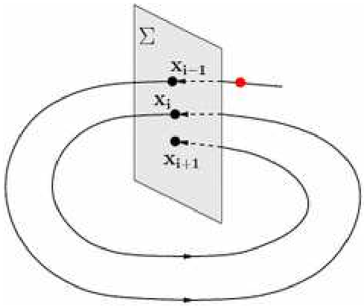
LIMIT SETS. THE POINCARÉ MAP

March 10, 12:30-15:15
Fredrik Bajers Vej 7D, room 2-109.

Lectures.

Aims and Content. What happens to a dynamical system "in the long run", what do orbits converge to for $t \to \pm\infty$? They may converge to an equilibrium point (sink or source), but they may also converge to a closed orbit. And for 2D-dynamical systems, essentially that's it; for higher dimensional systems, chaotic systems allow for much more intricate attractors.

The aim of these lectures is to introduce concepts and tools for an analysis of convergence of orbits. First, we need to know about α- and ω- limit sets and their properties (where it all begins/ends!) To analyse the behaviour of orbits in the neighbourhood of a closed orbit, the Poincaré map on a section perpendicular to the orbit is introduced and investigated. It associates to a point on this section the "first return point" on that section. By iterating the Poincaré map, one may extract information on the original continuous dynamical system from information about the discrete dynamical system given by the Poincaré map. In the 2D-case, the size of the derivative of the Poincaré map allows (often) to decide whether the closed orbit is asymptotically stable (or unstable).



Lecturer: Martin Raussen

References:

HSD: ch. 10.1-3.
Wikipedia: Limit set
Wikipedia: Poncaré map

Exercises:

Gradient/Hamiltonian: [HSD], p. 213, exc. 12-14.
Limit Sets: [HSD], p. 231-2, exc. 1.

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