LISBETH FAJSTRUP
MARTIN RAUSSEN
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MARCH 12, 2008

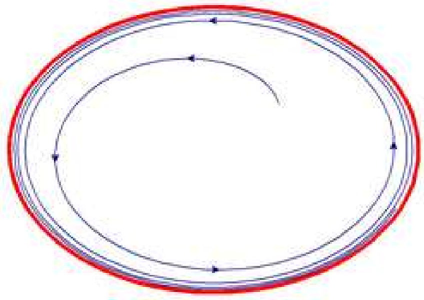
POINCARÉ BENDIXSON AND APPLICATIONS.

March 12, 9-11:45
Fredrik Bajers Vej 7D, room 2-109.

Lectures.

Aims and Content. Limit sets of planar systems are in the very focus during the lecture. We will confirm that chaos can not take place in planar dynamical systems.

A limit cycle is a closed orbit, such that some solution curve spirals toward it. It will be shown that if a closed and bounded (compact) set on $\mathbb{R}^2$ is positively or negatively invariant - the solution curve stays in it for positive or negative time - then it contains either a limit cycle or an equilibrium point. This is a consequence of the celebrated Poincare-Bendixson theorem.



Lecturer: Rafal Wisniewski

References:

HSD: 10.4-10.6
Wikipedia: Poincaré-Bendixson theorem

Exercises:

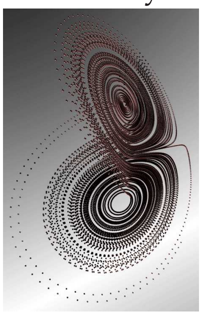
- p.234 Ex. 14

CHAOS.

March 12, 12:30 – 15:15

Lectures.

Aims and Content. The Lorenz system is classically known for the butterfly effect: If a butterfly flaps its wings in Brazil, it may cause a tornado in Texas. There are many definitions of chaos in a dynamical system. All of them will include “sensitive dependence on initial conditions initial conditions” in some version, i.e., that a slight change in initial conditions may cause a very large difference in the corresponding solution. Moreover, for chaos, we will usually require some kind of recurrence, which is best understood in terms of the Poincare map $g : D \to D$ around an *attractor*. An attractor A is a set which is invariant under the flow, and which has a basin of attraction, i.e. a larger set B such that solutions initiating in B will tend to A. A stable equilibrium is an example of an attractor, and an attracting cycle in the plane is another, but we will consider more complicated attractors. If the Poincare map has sensitive dependence of initial conditions, its periodic points are dense and it is transitive, then the attractor is a chaotic attractor. These definitions are best explained via an example, and we will study the Lorenz system as such an example.



It was actually unknown for a long time whether the system originally considered

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by Lorentz did contain a strange attractor. However, in 1999, a young Swedish PhD-student, Warwick Tucker managed to prove this.

Lecturer: Lisbeth Fajstrup

References:

HSD: 14.1-14.5 – read it "lightly" – skip the difficult parts...
Nature: Vol 406 p948-949
Wikipedia: Chaos theory

Exercises: The systems considered in the course have all been autonomous, i.e., the vector field does not depend on time. This exercise will illustrate some of the complications involved for non-autonomous systems and in particular, it serves as a warning - methods which work for autonomous systems could be misleading for non-autonomous cases.

We study the system $x' = -x + t$

(1) Find the solution to the system with initial condition $x(0) = x_0$
(2) Prove that all solution curves approach $t - 1$ for $t \to \infty$ and plot some solution curves in an (x, t) coordinate system.
(3) Define the "frozen time" or instantaneous fixed points for a system $x' = f(x, t)$ as points (x, t) where $f(x, t) = 0$. Find these for the above system. And plot them.
(4) Are the frozen time fixed points solutions to the differential equation?
(5) Plot the vector field for a fixed t. What information does this give you? Why does it seem that curves move away from the line $x = t - 1$ on the right hand side of it?

Exploration. As part of the evaluation of the course, you are to hand in the exploration 10.7 from [HSD]; you may of course collaborate in small groups as with the first exploration.

Deadline: Wednesday March 24.

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