Discussion of "Residual analysis for spatial point processes"

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To me, this great paper is a proof that statistics for spatial point processes has grown up into an adult branch of modern statistics. Now we can not only fit complex models to spatial point patterns but also assess the fitted models with tools analogous to the ones used for model assessment in the major fields of generalized linear models and survival analysis. My specific comments pertain to the lurking variable plots, residuals for Cox processes, and the K-function.

The lurking variable plot seems to some extent superfluous given that fitting a model including the variable is now routine using the eminent `R` library `spatstat`. Another potential use of a lurking variable type plot, however, would be to identify the correct scoring of a covariate. In survival analysis, for example, it has been proposed to use martingale residuals to identify the proper scoring of a covariate.

For many types of Cox and cluster processes including inhomogeneous log Gaussian Cox processes and certain Neyman-Scott processes, the intensity function is known in closed form whereas the conditional intensity is not. Hence, for such models it is more natural to construct residuals using the intensity. That is, letting $\lambda(\cdot)$ denote the intensity function, we might consider $n(X \cap B) - \int_B \lambda(u)\mathrm{d}u$ or $\sum_{x_i \in X \cap B} 1/\lambda(x_i) - |B|$ in analogy with the raw or inverse lambda innovation measures. Moreover, the second-order properties of the inverse intensity residuals are determined by the K-function as adapted to inhomogeneous point processes in Baddeley *et al.* (2000). Problems regarding the interpretation of the inhomogeneous case K-function occur when the intensity function is estimated nonparametrically. Waagepetersen (2005) in contrast considers a parametrized log linear intensity function for a certain class of inhomogeneous Neyman-Scott processes. In this paper I first obtain asymptotically normal estimates of the regression parameters and secondly use the consistently estimated intensity function to obtain a useful estimate of the K-function.

Returning to models specified in terms of a conditional intensity one might, inspired by Baddeley *et al.* (2000), define a K-type function by

$$\mathbb{E}\frac{1}{|B|}\sum_{x_i\in X\cap B, x_j\in X}\frac{1[0<\|x_i-x_j\|<t]}{\lambda(x_i,X)\lambda(x_j,X)}$$

This would characterise the second order moments of the inverse lambda innovation measure; for a pairwise interaction process with translation invariant interaction function c it has known expectation $\int_{\|v\|\le t} c(v)\mathrm{d}v$, see also (28).

References

Baddeley, A. J., Møller, J. & Waagepetersen, R. (2000). Non- and semi-parametric estimation of interaction in inhomogeneous point patterns. *Statistica Neerlandica* **54**, 329–350.

Waagepetersen, R. (2005). An estimating equation approach to inference for inhomogeneous Neyman-Scott processes. (in preparation).