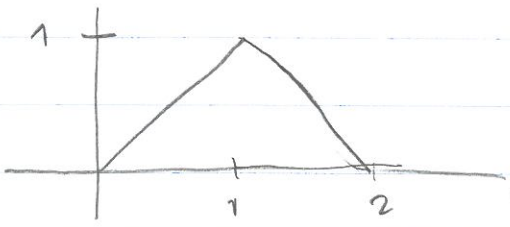
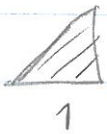
Opgaver 1 – facit

1

1)



2) $f(x) \geq 0$ ✓ $\qquad \int_0^2 f(x)\,dx = \frac{1}{2} \cdot 1 \cdot 2 = 1$ ✓

3) $P(X \leq 1) =$ areal  $1 = \frac{1}{2}$.

4)

Udregning af $F(x)$:

$0 \leq x \leq 1$: $\quad F(x) = \int_0^x f(z)\,dz = \int_0^x z\,dz = \frac{1}{2}x^2$

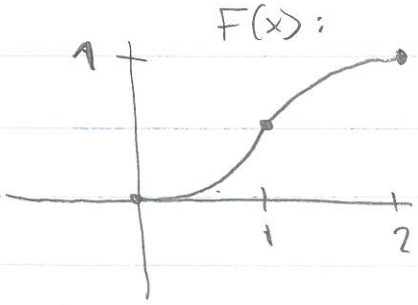
$1 < x \leq 2$: $\quad F(x) = \int_0^1 f(z)\,dz + \int_1^x f(z)\,dz =$

$\frac{1}{2} + \int_1^x (2-z)\,dz = \frac{1}{2} + \left[2z - \frac{1}{2}z^2\right]_1^x$

$= \frac{1}{2} + 2x - \frac{1}{2}x^2 - 2 + \frac{1}{2}$

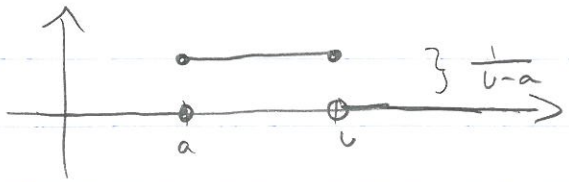
$= -\frac{1}{2}x^2 + 2x - 1$

$F(x)$:



2

1)



2) $f(x) \geq 0$ ✓ $\quad \int_{-\infty}^{\infty} f(x)\,dx = (b-a)\,\frac{1}{b-a}$

$= 1.$

3

1) $\mu = \int_a^b \frac{x}{b-a}\,dx = \frac{1}{b-a}\,\frac{1}{2}\left[b^2 - a^2\right] = \frac{a+b}{2}$

2) $\sigma^2 = EX^2 - \mu^2$

$$EX^2 = \int_a^b \frac{x^2}{b-a}\,dx = \frac{1}{b-a}\,\frac{1}{3}\left[x^3\right]_a^b$$

$$= \frac{1}{(b-a)3}\left(b^3 - a^3\right) = \frac{1}{3(b-a)}\,(b-a)\left[(a+b)^2 - ab\right] = \frac{1}{3}\left[(a+b)^2 - ab\right]$$

$$\sigma^2 = \frac{1}{3}(a+b)^2 - \frac{1}{3}ab - \frac{(a+b)^2}{4} = \frac{(b-a)^2}{12}$$

4

$$EY = 7 + 2.1 \cdot 2.8 = 12.88$$

$$Var\, Y = 2.1^2 \cdot 0.7 = 3.087$$

5

$$D = 750 + 200X$$

$$ED = 750 + 200\ 4.2 = 1590$$

$$Var\, D = 200^2 \cdot (1.4)^2 = 78400$$

$$\sigma_D = 200 \cdot 1.4 = 280$$

6

0.7257 | 0.9332 (*) | 0.8186 (**)

(* $1 - P(X \leq -1.5)$)

(** $P(X \leq 2) - P(X \leq -1)$)

7 | 0.7881 | 0.8849 | 0.8186

8 | 697 | 148 | 5 | 0

9 | 0.3585 | -0.3585 | 0.7722 | 0.9512 (*)

10 | 5.075 | $[0.5490;\ 7.450]$ □

[* $P(0.2 \leq X \leq x) = 0.25 \Rightarrow P(X \leq x) = 0.25 + P(X \leq 0.2)$] [□: find $x_{0.875}$ or $x_{0.125}$!]