

Facit - opgaver til lektion 2

1 a) $a = 1.96 \sigma = 1.96 \sqrt{0.04} = 1.96 \cdot 0.2 = 0.392$

b) $\frac{X+Y}{2} = \frac{1}{2}X + \frac{1}{2}Y \sim N\left(10, \frac{1}{2} \cdot 0.04\right)$

c) $b = 1.96 \sigma_{\frac{X+Y}{2}} = 1.96 \sqrt{\frac{1}{2} \cdot 0.04} = 1.96 \sqrt{\frac{1}{2}} \cdot 0.2$
 $= \sqrt{\frac{1}{2}} a = 0.2772$

Intervaller i c) er $\sqrt{\frac{1}{2}}$ smalle
end i a) fordi vi tager gennemsnit.

2

$\text{Var } U = 2\sigma^2$ $\text{Var } W = \text{Var}(X+X) = 4\sigma^2$

3

a) $\bar{x} = 1$

b) $\frac{\sigma^2}{n} = \frac{0.2^2}{5} = 0.008$

c) $1 \pm 1.96 \frac{0.2}{\sqrt{5}} = [0.8247; 1.1753]$

d) $1 \pm 2.58 \frac{0.2}{\sqrt{5}} = [0.7692; 1.2308]$

4 En måling $X_i \sim N(\beta, \sigma^2)$

$$a) \quad E\bar{X} = \beta \quad \text{Var}\bar{X} = \frac{\sigma^2}{n}$$

\bar{X} er normalfordelt

$$b) \quad P(|\bar{X} - \beta| < 3 \text{ mm}) = 0.95$$

$$\text{hvis} \quad 3 = 1.96 \frac{\sigma}{\sqrt{n}} \Leftrightarrow \sqrt{n} = \frac{1.96 \cdot \sigma}{3} \Leftrightarrow$$

$$n = \left(\frac{1.96 \cdot \sigma}{3} \right)^2 = 10.67 \approx 11$$