

Logistisk differentiaalligning

$$\begin{aligned}\frac{dy}{dt} &= k\left(1 - \frac{y}{A}\right)y \Leftrightarrow \frac{1}{k\left(1 - \frac{y}{A}\right)y} dy = 1 dt \Leftrightarrow \frac{1}{k} \int \frac{1}{\left(1 - \frac{y}{A}\right)y} dy = \int 1 dt \\ \Leftrightarrow \int \frac{1}{\left(1 - \frac{y}{A}\right)y} dy &= k \int 1 dt \Leftrightarrow \int \frac{1}{y} + \frac{\frac{1}{A}}{1 - \frac{y}{A}} dy = k \int 1 dt \Leftrightarrow \\ \ln y - \ln\left(1 - \frac{y}{A}\right) &= kt + c \Leftrightarrow \ln \frac{y}{1 - y/A} = kt + c \Leftrightarrow \\ \frac{y}{1 - y/A} &= B \exp(kt) \Leftrightarrow y = B \exp(kt) - \frac{B}{A} y \exp(kt) \Leftrightarrow \\ y &= \frac{B \exp(kt)}{1 + \frac{B}{A} \exp(kt)} \Leftrightarrow y = \frac{A}{\frac{A}{B} \exp(-kt) + 1}\end{aligned}$$

Dvs. løsningskurve er en logistisk vækstfunktion.