Opgave 15.3.3

$$\frac{dy}{dt} + 2ty = t \quad ; \quad y(0) = \frac{3}{2}$$

Bruger generel løsningsformel med $u(t) = 2t$ og $w(t) = t$

$$y(t) = \exp\left(-\int 2t\,dt\right)\left(A + \int t \exp\left(\int 2t\,dt\right) dt\right)$$

$$= \exp(-t^2 - k)\left(A + \int t \exp(t^2 + k)\,dt\right)$$

$$= \exp(-t^2 - k)A + \exp(-t^2 - k)\exp(t^2 + k)\frac{1}{2}$$

$$= B\exp(-t^2) + \frac{1}{2} \qquad (B = A\exp(-k))$$

Check: $y'(t) = -2t\,B\exp(-t^2)$.

$$\frac{dy}{dt} + 2ty = -2tB\exp(-t^2) + 2t\left(B\exp(-t^2) + \frac{1}{2}\right)$$

$$= 0 + \frac{2t}{2} = t \quad \checkmark$$

Begyndelsesbetingelsen giver $y(0) = \frac{3}{2} \Leftrightarrow B + \frac{1}{2} = \frac{3}{2}$

$$\Leftrightarrow B = 1.$$

Dvs den fuldstændige løsning er $\underline{y(t) = \exp(-t^2) + \frac{1}{2}}$