Forventet fortjeneste : news-vendor modellen

D (efterspørgsel) har tæthed $f(x) = \frac{1}{80-50} = \frac{1}{30}$ for $x \in [50, 80]$

$$EX = p\int_{-\infty}^{q} x f(x)\,dx + pq\left(1 - F(q)\right) - cq$$

$$F(q) = \int_{50}^{q} f(x)\,dx = \int_{50}^{q} \frac{1}{30}\,dx = \frac{q-50}{30} \quad \text{for } q \in [50, 80]$$

(Bemærk $F(50) = 0$, $F(80) = 1$)

$$\int_{-\infty}^{q} x f(x)\,dx = \int_{50}^{q} \frac{x}{30}\,dx = \frac{1}{30}\left[\frac{1}{2}x^2\right]_{50}^{q}$$

$$= \frac{1}{60}\left(q^2 - 2500\right)$$

Dvs

$$EX = \frac{p}{60}\left(q^2 - 2500\right) + pq\left(1 - \frac{q-50}{30}\right) - cq$$

$$= \frac{pq^2}{60} - \frac{p \cdot 250}{6} + pq - \frac{pq^2 - pq\,50}{30} - cq$$

$$= q^2\left(\frac{p}{60} - \frac{p}{30}\right) + q\left(p + p\,\frac{50}{30} - c\right) - \frac{p\,250}{6}$$

$$= -q^2\,\frac{p}{60} + q\left(p\left(1 + \frac{5}{3}\right) - c\right) - p\,\frac{250}{6}$$

Dvs 2. gradspolynomium, der vender grenene nedad.

Toppunkt:

$$-2q\frac{p}{60} + p\left(1+\frac{5}{3}\right) - c = 0 \iff$$

$$q\frac{p}{30} = p\left(1+\frac{5}{3}\right) - c \iff q = 30\left(1+\frac{5}{3}\right) - 30\frac{c}{p}$$

$$= 80 - 30\frac{c}{p}$$

Sammenligning med tidligere resultat:

Optimalt $q = F^{-1}\left(\frac{p-c}{p}\right) = F^{-1}\left(1-\frac{c}{p}\right)$

$$y = F(x) \iff y = \frac{x-50}{30} \iff x = 30y + 50$$

Dvs $F^{-1}(y) = 30y + 50$

Dermed $q = 30\left(1-\frac{c}{p}\right) + 50 = 80 - 30\frac{c}{p}.$