Retteviledning 5. lektion

16.3.1

$y'' - 4y' + 8y = 0 \qquad y(0) = 3 \qquad y'(0) = 7$

Karakteristisk ligning:

$r^2 - 4r + 8 = 0$, $\quad D = 16 - 4 \cdot 8 = -16$ dvs rødder er komplekse.

Rødder $\quad r_1 = \dfrac{4 + \sqrt{-16}}{2} = \dfrac{4 + i4}{2} = \underbrace{2}_{h} + i\underbrace{2}_{v}$

$r_2 = 2 - i2$

Bemærk at ligningen er homogen (højresiden $= 0$) dvs skal ikke finde partikulær løsning.

Dermed er løsningen på formen 16.24': $y = e^{2t}\left[A_5 \cos 2t + A_6 \sin 2t\right]$

Benytter begyndelsesbetingelser til at finde A_5 og A_6:

$y(0) = 3$ og $y(0) = 1 \cdot A_5 \cdot 1 + 1 \cdot A_6 \cdot 0 = A_5$ dvs $A_5 = 3$

$y'(t) = 2e^{2t}\left[A_5 \cos 2t + A_6 \sin 2t\right] + e^{2t}\left[-A_5 2\sin 2t + 2A_6 \cos 2t\right]$

$y'(0) = 2A_5 + 2A_6 = 7$ og $A_5 = 3$ dvs $2A_6 = 1 \Leftrightarrow A_6 = \frac{1}{2}$

Omskriver vha additionsformel 16.16:

$(A_S, A_C) = \underbrace{(A\cos\varepsilon, A\sin\varepsilon)}$ $\quad \varepsilon = \operatorname{atan}\frac{A_S}{A_C}$ $\quad A = \sqrt{A_S^2 + A_C^2}$

(A, ε) polare koordinater for (A_S, A_C).

$$\varepsilon = \operatorname{atan}\left(\frac{1/2}{3}\right) = \operatorname{atan}\left(\frac{1}{6}\right) = 0.165 \qquad A = 3.04$$

$$y = e^{2t}(3\cos 2t + 0.5\sin 2t) =$$

$$3.04\, e^{2t}\left[\cos 0.165 \cdot \cos 2t + \sin 0.165 \cdot \sin 2t\right] = 3.04\, e^{2t}\cos(2t - 0.165)$$

Check

$$y' = 2 \cdot 3.04\, e^{2t}\cos(2t-0.165) - 2 \cdot 3.04\, e^{2t}\sin(2t-0.165)$$

$$y'' = \underbrace{2\cdot 2 \cdot 3.04\, e^{2t}\cos(2t-0.165) - 2\cdot 2\cdot 3.04 e^{2t}\sin(2t-0.165)}$$
$$- 2\cdot 2\cdot 3.04\, e^{2t}\sin(2t-0.165) \underbrace{- 2\cdot 2\cdot 3.04 e^{2t}\cos(2t-0.165)}$$

$$= -8 \cdot 3.04 e^{2t}\sin(2t-0.165)$$

$$y'' - 4y' + 8y = -8\cdot 3.04 e^{2t}\sin(2t-0.165) - 8\cdot 3.04\, e^{2t}\cos(2t-0.165) +$$
$$8\cdot 3.04\, e^{2t}\sin(2t-0.165) + 8\cdot 3.04 e^{2t}\cos(2t-0.165)$$

$$= 0 \ \checkmark$$

$$y(0) = 3.04 \cdot \cos(-0.165) = 3 \ \checkmark$$

$$y'(0) = 2\cdot 3.04\cos(-0.165) - 2\cdot 3.04\sin(-0.165) = 7 \ \checkmark$$