Exercises on Monte Carlo Methods

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Suppose $U \sim N(0,1)$ and $Y|U = u \sim \text{Poisson}(\exp(\beta + U))$ (recall that Poisson(λ) has density $f(y;\lambda) = \exp(-\lambda)\lambda^y/y!$). Assume that $Y = 8$ is observed.

1. Compute and plot a simple Monte Carlo approximation of the likelihood for β values over the interval $[-1, 4]$ and compute estimates of the Monte Carlo error. Compare results obtained with different numbers of simulations. Consider also the case where 10 observations 8, 18, 5, 7, 10, 9, 9, 6, 7, 10 are available (i.e. $f(8; \exp(\beta + u)$ is replaced by the product

$$\prod_{i=1}^{10} f(y_i; \exp(\beta + u))$$

of conditional densities for these observations).

2. This exercise considers importance sampling approximation of the likelihood using a t-distribution with mean and variance from the Laplace approximation obtained in the previous exercises.

   (a) In the same plot draw the density of the importance t-distribution and of the joint density

$$\exp(g(u)) = \prod_{i=1}^{10} [f(y_i; \exp(\beta + u))/f(y_i; \bar{y})]f(u)$$

of the observations and the random effect (here we have for numerical stability divided by $f(y_i; \bar{y})$. How does the importance sampling distribution depend on the value of β ?

(b) Compute an importance sampling approximation of the likelihood. What happens if you use the importance t-distribution obtained for, say $\beta = 4$, to compute the likelihood for other values of β in the interval $[-1, 4]$?