Exercises - computation of the likelihood for a GLMM

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You may either program the solutions from scratch or consider the programs in the `.R` file `exercise_lp_gh.R`. However, to encourage you to work actively with the code you will often need to either first correct bugs in the programs or modify the programs before you get the correct solution.

Suppose $U \sim N(0,1)$ and $Y|U = u \sim \text{Poisson}(\exp(\beta + U))$ (recall that $\text{Poisson}(\lambda)$ has density $f(y;\lambda) = \exp(-\lambda)\lambda^y/y!$). Assume that $Y = 8$ is observed.

a) In the same plot draw the density $f(u) = \exp(-u^2/2)/\sqrt{2\pi}$ of $N(0,1)$ and $f(8; \exp(\beta + u))$ as a function of u and for various β-values. Also plot the product $f(8; \exp(\beta+u))f(u)$ of the two densities. Can you tell which values of β yield small likelihoods ?
b) Compute and plot the marginal likelihood of β using numerical quadrature (use the `R` function `integrate`).
c) Compute and plot a Laplace-approximation of the likelihood (note that you obtain as a biproduct a normal distribution with mean and variance approximately equal to the conditional mean and variance of U given $Y = y$).
e) compute the likelihood using adaptive Gaussian quadrature (GH-schemes are available on the webpage).
f) what is the MLE of β ?
g) compare the conditional density of $U|Y = y$ and the approximating normal density obtained from the Laplace approximation at the MLE.

h) Repeat a) - c) but in the situation where 10 observations 8, 18, 5, 7, 10, 9, 9, 6, 7, 10 are available (i.e. $f(8; \exp(\beta + u))$ is replaced by the product

$$\prod_{i=1}^{10} [f(y_i; \exp(\beta + u))$$

of conditional densities for these observations). Note that we now have to take care of numerical problems since the product of 10 Poisson densities may take very small values. One possibility is to stabilize by dividing with the product of Poisson densities evaluated at $\hat{\lambda} = \bar{y}$ (the MLE). Does the accuracy of the Laplace approximation improve or worsen when more observations are available ?