Statistics.
Comments to exercises 1. lecture

Calculations are aided by python.

Exercise [EL] 1

(a)

$$P(X \leq 30) \quad = \quad \Phi\left(\frac{30-30}{0.05}\right) = \Phi(0) = \texttt{norm..cdf}(0) = 0.5.$$

(b)

$$P(X \geq 29.9) \quad = \quad 1 - \Phi\left(\frac{29.9-30}{0.05}\right) = 1 - \Phi(-2) = 1 - \texttt{norm..cdf}(-2) = 0.977$$

(c)

$$\begin{aligned} P(29.9 \leq X \leq 30.15) \quad &= \quad P(X \leq 30.15) - P(X \leq 29.9) = \Phi\left(\frac{30.15-30}{0.05}\right) - (1 - 0.977) = \Phi(3) \\ &= \quad \texttt{norm.cdf}(3) - 0.023 = 0.976 \end{aligned}$$

Exercise [EL] 2

(a) The probability of a single measurement above $30m$ is $1/2$. So the answer is

$$(1/2)^4 = 1/16 = 0.0625.$$

(b) $P(X \leq 29.9) = 0.023$. So the answer is

$$(.023)^4 = 2.8e - 07$$

(c) This requires the complementaty event: $P(A) = 1 - P(\neg A)$. So

$$1 - \Phi\left(\frac{30.15-30}{0.05}\right)^4 = 1 - \texttt{norm..cdf}(3)\texttt{\^{}}4 = 0.00539.$$

Exercise [EL] 3

Solution in script file exe3.py.

The values of the variables should be close to $a = 63$, $b = 0$, and $c = 5$, but of course the results vary due to randomness.

Exercise [EL] 4

(a)

$$1000\left[2\left\{1-\Phi\left(\frac{7.5}{5}\right)\right\}\right] = 2000 * (1 - \texttt{norm.cdf}(1.5)) = 133.6$$

(b)

$$1000\left[2\left\{1-\Phi\left(\frac{15}{5}\right)\right\}\right] = 2000 * (1 - \texttt{norm.cdf}(3)) = 2.7$$

Exercise [EL] 6

(a) A priori the variance is 1. A posteriori the variance multiplied by the number d of redundants is χ^2 with d degrees of freedom. Hence

$$\begin{aligned} P(0.8 \leq S^2 \leq 1.2) &= P(16 \leq 20S^2 \leq 24) \\ &= \texttt{chi2.cdf}(24, 20) - \texttt{chi2.cdf}(16, 20) = 0.4742. \end{aligned}$$

(b) Similarly

$$\texttt{chi2.cdf}(12, 10) - \texttt{chi2.cdf}(8, 10) = 0.3438.$$