Written exam in Probability Theory and Statistics - K7

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Thursday 13th of january 2011, 9:00-13:00
In the assessment emphasis will be put on both correct methods as well as correct answers. Hence the method should be clearly stated.

Problem 1. (approx 20%)
The random variable X has a normal distribution with mean 4 and variance 25. 1. Mean: $4E(X)+6 = 4\cdot 4+6 = 22$, Var: $16\cdot\sigma_X^2 = 400$

1. Calculate the mean and variance of the variable $4X + 6$.
2. Calculate $P(0 \le X \le 4)$. $= P(X \ge 0) - P(X \ge 4) = 0.5 - 0.212 = 0.288$

The random variable Y has mean 5 and variance 10. The correlation coefficient of X and Y is -0.5. Mean: $4E(X) + 5E(Y) + 1 = 4\cdot 4 + 5\cdot 5 + 1 = 42$

3. Calculate the mean and variance of the variable $4X + 5Y + 1$. Var: $4^2\sigma_X^2 + 5^2\sigma_Y^2 + 2\cdot 4\cdot 5\,\sigma_{XY} =$
$16\cdot 25 + 25\cdot 10 + 40\times(-0.5\cdot\sqrt{25\cdot 10})$
≈ 333.8

Dist of X:

$x \backslash y$	0	1	2	SUM
0	0	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{3}{27}$
1	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{9}{27}$
2	$\frac{4}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{15}{27}$

Problem 2. (approx 20%)
The joint probability distribution of X and Y is given by

$$f(x, y) = \frac{2x + y}{27}, \quad x = 0, 1, 2; \; y = 0, 1, 2$$

1. Evaluate the marginal distribution of X. $P(Y=2 \mid X=1) = \frac{f(1,2)}{P(X=1)} = \frac{4}{9}$
2. Find $P(Y = 2|X = 1)$ and $P(Y = 2|X = 2)$. Are X and Y statistically independent? $P(Y=2 \mid X=2) = f(2,2)/P(X=2) = \frac{6}{15}$
3. Evaluate $E(X^2Y)$. $\sum_{x=0}^{2}\sum_{y=0}^{2} x^2 y\, f(x,y) = 1\cdot\frac{3}{27} + 2\cdot\frac{4}{27} + 4\cdot\frac{5}{27} + 8\cdot\frac{6}{27} = \frac{79}{27}$

No:
$P(Y=2 \mid X=1) \neq P(Y=2 \mid X=2)$

Problem 3. (approx 10%)
In a certain city the need for money to buy drugs is stated as the reason for 60% of all thefts.
Consider the next 20 theft cases in the city and let X denote the number of cases resulting from the need for money to buy drugs.

1. Calculate the mean and variance of X. $\sim$ binomial $(n=20, p=0.6)$
2. Evaluate $P(4 \le X \le 12)$. $EX = n\cdot p = 12$ $\quad \sigma_X^2 = n\cdot p(1-p) = 4.8$

$= P(X \le 12) - P(X \le 3) \approx 0.548$

Problem 4. (approx 30%)
An engineer in quality control takes a sample of 30 bolts and measures their diameter, which yields a sample average of $\bar{x} = 10.023mm$ and a sample standard deviation $s = 0.009mm$. He assumes that the observations are a random sample from the normal distribution.

Walpole P. 279

$\nu = 29$, $t_{\alpha/2} = 2.045$

$\rightarrow [10.01964, 10.02635]$

1. Determine a 95% confidence interval for the mean of the bolt diameter.
2. Determine a 95% confidence interval for the standard deviation of the bolt diameter.

Walpole p. 307:

$\nu = 29$,

$\chi^2_{\alpha/2} = 16.05$

$\chi^2_{1-\alpha/2} = 45.72 \rightarrow$

$[0.514, 1.464] \times 10^{-4}$

$\sqrt{\ } \rightarrow [0.717, 1.210] \times 10^{-2}$

3. Test at the 5% significance level whether the bolts meet a requirement of a mean diameter equal to $10mm$.
4. Test at the 2.5% significance level whether the measurements meet a requirement of a standard deviation below or equal to $0.005mm$.

Not in confidence interval $\Rightarrow$ reject.

Problem 5. (approx 20%)
Two methods for measuring the molar heat of fusion of water are being compared. Ten measurements made by method A have a sample mean $\bar{x}_A = 6.025$ kilojoules per mole and sample standard deviation of $s_A = 0.024KJ/mol$. Five measurements made by method B have a sample mean $\bar{x}_B = 6.001KJ/mol$ and sample standard deviation of $s_B = 0.012KJ/mol$.

1. Test at the 5% significance level whether the two methods have the same standard deviation.

Walpole p 368

$F = \left(\frac{24}{12}\right)^2 = 4$

$\nu_1 = 9 \quad \nu_2 = 4$

$f_{\alpha/2}(9,4) = 8.9 > 4$

Do not reject

2. Test at the 5% significance level whether the mean measurements differ between the two methods.

Remember to add student number on all sheets and state how many sheets your solution consists of

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Walpole p. 346: $s_p^2 = 4.43 \times 10^{-4}$

$t = 2.082 < t_{\alpha/2, 13} = 2.16$

Do not reject.