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## Written exam in Probability & Statistics

PM6 & ET6

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Friday 6th of January 2006, 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

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### Problem 1. (approx. 20%)

A salesman at a used car dealer receives a commission for each car or van he sells. When he sells a car he receives 4200 kr and 4800 kr when he sells a van. He expects to sell a number of cars and vans each day according to the following probabilities:

|                |  |     |     |     |     |
|----------------|--|-----|-----|-----|-----|
| Number of cars |  | 0   | 1   | 2   | 3   |
| Probability    |  | 0.3 | 0.4 | 0.2 | 0.1 |

|                |  |     |     |     |
|----------------|--|-----|-----|-----|
| Number of vans |  | 0   | 1   | 2   |
| Probability    |  | 0.4 | 0.5 | 0.1 |

1. Calculate the expected number of cars and vans the salesman is expected to sell each day.
2. Calculate the standard deviation of the number of cars and van the salesman sells in a day.
3. Calculate the expected commission for both cars and vans a salesman will receive in a day.
4. Calculate the standard deviation of the salesman total commission in a day when we assume that the number of sold cars and sold vans are dependent with a correlation coefficient of  $\rho = 0.1$ .

### Problem 2. (approx. 15%)

The length of times it takes to repair a vending machine follows a normal distribution with mean 120 minutes and variance 16 minutes<sup>2</sup>. If the vending machine is under repair for more than 125 minutes the machines must be cleaned and emptied which is an unwanted extra expense.

1. What is the probability that the vending machine is under repair for more than 125 minutes?
2. A member of staff wants to find a time interval in which the time it takes to repair the vending machine is with 95% probability. Find such a 95% probability interval which is symmetric around the mean.

**Problem 3.** (approx. 15%)

Wanting to optimise storage space a seller wants to model the number of orders on a specific product in December. In December the previous year the number of orders was 15.

1. Specify a random variable and its distribution, so that it describes that number of orders in December — explain your choice.
2. What is the probability of 17 or more orders.
3. How large does stock need to be for the seller to have at least a 95% probability of fulfilling all orders? Assume that the seller cannot receive new stock during December.

**Problem 4.** (approx. 30%)

The walls in a plastic bottle need to have a certain thickness to avoid that the bottle does breaks. An engineer in quality control takes a sample of 25 bottles and measures the wall thickness obtaining a sample average of  $\bar{x} = 4.05mm$  and a sample standard deviation of  $s = 0.08mm$ . He further assumes that the observations are independent and normal distributed.

1. Determine a 95% confidence interval for the mean of the wall thickness.
2. Determine a 95% confidence interval for the standard deviation of the wall thickness.
3. Test at the 5% significance level if the wall thickness is less than  $4mm$ .
4. Test at the 5% significance level if the standard deviation of the wall thickness equals 0.1

**Problem 5.** (approx. 20%)

A cement factory wants to buy a new machine for filling bags with 50kg of cement. They have two machines to choose from. From each machine they take a sample of 6 bags and weigh each of them. The measured weight are given in the table below

|           | Machine I | Machine II |
|-----------|-----------|------------|
|           | 51.2      | 29.4       |
|           | 49.0      | 50.7       |
|           | 49.8      | 49.1       |
|           | 51.7      | 48.7       |
|           | 50.3      | 48.7       |
|           | 51.4      | 50.1       |
| $\bar{x}$ | 50.57     | 49.80      |
| $s^2$     | 1.0987    | 0.7520     |

1. Test at the 10% significance level if the variance of the weights are equal for the two machines.
2. Test at the 10% significance level if the means of the weights are equal for the two machines.

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Remember to add student number on all sheets and state how many sheets your solution consists of.

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