Written exam in Probability & Statistics

PM6 & ET6
Lecturer: Kasper K. Berthelsen

Tuesday 4th of January 2005, 9:00-13:00

In the assessment emphasis will be put on both correct methods as wells as correct answer, hence the the method should be clearly stated.

Good luck!

Problem 1. (approx. 20%)
A random variable X has mean 10 and variance 50.

1. Calculate the mean and variance of the random variable $Y = 10 + 5X$.
2. Find the mean of $V = (X - 10)^2$ and $Z = X^2$.

Problem 2. (approx. 10%)
In a survey among house owner, people are asked if they are willing to pay more for snow removal. Among the 84 people who reply the answers are distributed according to age as follow:

Age	No	Yes	No answer
20–25	1	0	0
26–35	0	3	1
36–50	6	10	10
51–60	1	7	1
61–70	2	13	6
> 70	4	13	6

1. Calculate the conditional probability for answering yes, conditionally on the person's age being ≤ 50 years and > 50 years, respectively.
2. Are the events $A = \{\text{Age} \leq 50 \text{ years}\}$ and $B = \{\text{Yes}\}$ independent? Justify your answer.

Problem 3. (approx. 20%)
In an airport, whenever the metal detector goes off, there is a 25% probability that the alarm is caused by coins in the pocket of the passenger walking through the metal detector.

1. During one day the alarm goes off 15 times. What is the probability that at least 3 of these alrams are caused by passengers having coins in their pockets?
2. Question 1 continued: Is it likely that none of these 15 alarms are caused by coins in a pocket? Explain your answer based on the probability of this event.
3. Just before Christmas the airport is unusually busy. On one day the metal detector alarm goes off 50 times. What is the probability that at most $\frac{1}{5}$ of these alarms are caused by coins in a pocket.

Problem 4. (approx. 35%)
As is well-known, the department network is often down. Near the project dead-line some students decide to measure the dayly downtime in minutes. Acordingly they measure how many minutes the network is down each day for 14 days and obtain the following downtimes:

Day	1	2	3	4	5	6	7
downtime (minutes)	229	295	343	337	282	313	262
Day	8	9	10	11	12	13	14
downtime (minutes)	303	201	374	376	343	406	163

The downtimes are assumed to follow a normal distribution with mean μ and variance σ^2.

1. Estimate the mean μ and the standard deviation σ for the dayly downtime.
2. Determine a 95% confidence interval for μ.
3. Determine a 95% confidence interval for σ.
4. The students want the downtime to be as short as possible. Test on the 5% significance level if the expected downtime is significantly less than 4 hours, i.e. 240 minutes.
5. What is the probability that the average down time over a 14 day period is less then 4 hours, i.e. 240 minute? Assume that the 14 downtimes are independent and normal distributed with equal means $\mu = 300$ and unknown and equal variances.

Problem 5. (approx. 15%)
At a Christmas dinner 10 students measure their blood alcohol level by each making two measurements using a breathalyzer. They obtain the following measurements where the differences between measurements are given:

Student	1st measurement	2nd measurement	Difference
1	0.9	0.9	0.0
2	1.0	1.8	-0.8
3	1.8	1.8	0.0
4	1.2	1.6	-0.4
5	0.8	0.8	0.0
6	1.0	0.8	0.2
7	0.9	1.0	-0.1
8	1.2	2.1	-0.9
9	2.2	2.0	0.2
10	1.2	1.5	-0.3
$\bar{x}$	1.22	1.43	-0.21
s^2	0.197	0.260	0.150

It is assumed that the random variables correspoindg to the alchohol level for first and second measurments are independent and noraml distributed with equal mean and variance. Notice that the two measurements for the same student are **not** independent.

1. Find a 90% confidence interval for the difference in the two measurements.
2. Tes at the 5% significance level if the level at the first measurement is different from the secon measurement.

Remember to add student number on all sheets and state how many sheets your solution consists of.