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## Written exam in Probability Theory and Statistics - K7

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Thursday 13th of January 2011, 9:00-13:00

In the assessment emphasis will be put on both correct methods as well as correct answers. Hence the method should be clearly stated.

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### Problem 1. (approx 20%)

The random variable  $X$  has a normal distribution with mean 4 and variance 25.

1. Calculate the mean and variance of the variable  $4X + 6$ .
2. Calculate  $P(0 \leq X \leq 4)$ .

The random variable  $Y$  has mean 5 and variance 10. The correlation coefficient of  $X$  and  $Y$  is  $-0.5$ .

3. Calculate the mean and variance of the variable  $4X + 5Y + 1$ .

### Problem 2. (approx 20%)

The joint probability distribution of  $X$  and  $Y$  is given by

$$f(x, y) = \frac{2x + y}{27}, \quad x = 0, 1, 2; \quad y = 0, 1, 2$$

1. Evaluate the marginal distribution of  $X$ .
2. Find  $P(Y = 2|X = 1)$  and  $P(Y = 2|X = 2)$ . Are  $X$  and  $Y$  statistically independent?
3. Evaluate  $E(X^2Y)$ .

### Problem 3. (approx 10%)

In a certain city the need for money to buy drugs is stated as the reason for 60% of all thefts.

Consider the next 20 theft cases in the city and let  $X$  denote the number of cases resulting from the need for money to buy drugs.

1. Calculate the mean and variance of  $X$ .
2. Evaluate  $P(4 \leq X \leq 12)$ .

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**Problem 4.** (approx 30%)

An engineer in quality control takes a sample of 30 bolts and measures their diameter, which yields a sample average of  $\bar{x} = 10.023mm$  and a sample standard deviation  $s = 0.009mm$ . He assumes that the observations are a random sample from the normal distribution.

1. Determine a 95% confidence interval for the mean of the bolt diameter.
2. Determine a 95% confidence interval for the standard deviation of the bolt diameter.
3. Test at the 5% significance level whether the bolts meet a requirement of a mean diameter equal to  $10mm$ .
4. Test at the 2.5% significance level whether the measurements meet a requirement of a standard deviation below or equal to  $0.005mm$ .

**Problem 5.** (approx 20%)

Two methods for measuring the molar heat of fusion of water are being compared. Ten measurements made by method A have a sample mean  $\bar{x}_A = 6.025$  kilojoules per mole and sample standard deviation of  $s_A = 0.024KJ/mol$ . Five measurements made by method B have a sample mean  $\bar{x}_B = 6.001KJ/mol$  and sample standard deviation of  $s_B = 0.012KJ/mol$ .

1. Test at the 5% significance level whether the two methods have the same standard deviation.
2. Test at the 5% significance level whether the mean measurements differ between the two methods.

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Remember to add student number on all sheets and state how many sheets your solution consists of

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