Øvelse s17

Sekundteodolit: $\sigma_{R,1} = 8^{cc}$ (retningsspredning)

måler med to satser dvs. to vinkelmålinger: X_1 og X_2 uafh.,

hvor

$$\mathbb{E}(X_1) = \mathbb{E}(X_2) = \mu$$

$$\mathbb{V}(X_1) = \mathbb{V}(X_2) = 2\,\sigma_{R,1}^2 \qquad (\mathbb{V}(R_2 - R_1) = \mathbb{V}(R_2) + \mathbb{V}(R_1) = 2\,\sigma_R^2)$$

Middelsats: $\bar{X} = \frac{1}{2}(X_1 + X_2)$

$$\mathbb{V}(\bar{X}) = \left(\frac{1}{2}\right)^2 (\mathbb{V}(X_1) + \mathbb{V}(X_2)) = \frac{1}{4}(2\sigma_{R,1}^2 + 2\sigma_{R,2}^2) = \sigma_{R,1}^2$$

Mindre teodolit: $\tau_{R,2}^2 = 20^{cc}$ (retningsspredning)

måler med n satser, dvs. $Y_1, \ldots, Y_n$ uafh, hvor

$$\mathbb{E}(Y_i) = \mu$$

$$\mathbb{V}(Y_i) = 2 \cdot \tau_{R,2}^2$$

Middelsats: $\bar{Y} = \frac{1}{n}(Y_1 + \ldots + Y_n)$

$$\mathbb{V}(\bar{Y}) = \frac{1}{n^2}\, n\, \mathbb{V}(Y_i) = \frac{2\,\tau_{R,2}^2}{n}$$

Samme vægt af middelsatserne, dvs. ens varians af $\bar{X}$ og $\bar{Y}$, altså

$$\sigma_{R,1}^2 = \frac{2\,\tau_{R,2}^2}{n}$$

$$\Updownarrow$$

$$n = \frac{2\,\tau_{R,2}^2}{\sigma_{R,1}^2} = \frac{2 \cdot 20^2}{8^2} = 12.5$$

dvs. der skal måles med 13 satser for at begge middelsatser kan indgå med samme vægt.

Opgave 7.2 μ: geometrisk nivelleret over 3 strækninger $\sigma_k = \sigma_0 = 5{,}1\ \mathrm{mm}/\sqrt{\mathrm{km}}$

a)

Stræk-ning	ℓ (km)	h (mm)	vægt $p = \frac{1}{\ell}$ (km^{-1})	$\frac{p}{p_\cdot}$	$\frac{p}{p_\cdot} h$
1	1,142	9751	0.8757	0.4222	4117,0
2	1,583	9759	0.6317	0.3046	2972,6
3	1,765	9753	0.5666	0.2732	2664,4

$p_\cdot = 2{,}074$ $\quad \Sigma = 1$ $\quad \bar{x}^* = \underline{\underline{9754{,}0\ \mathrm{mm}}}$

b) Spredning på vægtet gennemsnit: $\frac{\sigma_0}{\sqrt{p_\cdot}} = \frac{5.1\ \mathrm{mm}/\sqrt{\mathrm{km}}}{\sqrt{2.074}\ 1/\sqrt{\mathrm{km}}} = \underline{\underline{3{,}54\ \mathrm{mm}}}$ ✓

c) $\bar{X}^* \sim N(\mu, \sigma^2)$ hvor $\sigma = 3.54$ mm

$$P\left(-1.96 \le \frac{\bar{X}^* - \mu}{\sigma} \le 1.96\right) = 0.95$$

Med 95 % ss. vil obs. af $\bar{x}^*$ ligge i

$$-1.96 \le \frac{\bar{x}^* - \mu}{\sigma} \le 1.96$$

$$\Updownarrow$$

$$\bar{x}^* - 1{,}96\sigma \le \mu \le \bar{x}^* + 1{,}96\sigma$$ 95 % konfidensinterval for μ

Indsæt $\bar{x}^* = 9754{,}0$ mm og $\sigma = 3.54$ mm

$[9747{,}1\ \mathrm{mm};\ 9760{,}9\ \mathrm{mm}]$ 95 % konfidensinterval for μ

KMS: $h = 9{,}75$ m stemmer overens med vores observationer

Opgave 7.1 Målt μ to gange aus X_1 og X_2 uafh

$$\mathbb{E}(X_1) = \mathbb{E}(X_2) = \mu$$

$$\mathbb{V}(X_1) = \sigma_1^2 < \mathbb{V}(X_2) = \sigma_2^2$$

Lad $0 < \alpha < 1$:

a) $\alpha X_1 + (1-\alpha) X_2$ central estimator, da

$$\mathbb{E}(\alpha X_1 + (1-\alpha) X_2) = \alpha \mathbb{E}(X_1) + (1-\alpha)\mathbb{E}(X_2) = \alpha\mu + (1-\alpha)\mu = \mu$$ ✓

b) $$\mathbb{V}(\alpha X_1 + (1-\alpha) X_2) = \alpha^2 \mathbb{V}(X_1) + (1-\alpha)^2 \mathbb{V}(X_2) = \alpha^2\sigma_1^2 + (1-\alpha)^2\sigma_2^2$$ ✓

$$\mathbb{V}(\alpha X_1 + (1-\alpha) X_2) = \alpha^2\sigma_1^2 + (1+\alpha^2-2\alpha)\sigma_2^2$$

$$= (\sigma_1^2+\sigma_2^2)\alpha^2 - 2\sigma_2^2\alpha + \sigma_2^2$$

$$= A\alpha^2 + B\alpha + C \qquad \min \alpha = \frac{-B}{2A}$$

dvs. $\min \alpha = \frac{+2\sigma_2^2}{2(\sigma_1^2+\sigma_2^2)} = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$ ✓

c) Antag $x_1 = 10$, $x_2 = 14$, $\sigma_1^2 = 4$ og $\sigma_2^2 = 8$

$$\alpha = \frac{8}{4+8} = \frac{8}{12} = \frac{2}{3}$$

$$\bar{x}^* = \frac{2}{3}x_1 + \frac{1}{3}x_2 = \frac{2}{3}10 + \frac{1}{3}14 = \frac{34}{3} = \underline{\underline{11\frac{1}{3}}}$$